Journal of Sound and Vibration (2001) **246**(1), 185–189
doi:10.1006/jsvi.2001.3581, available online at http://www.idealibrary.com on IDEAL®

A SIMPLE FORMULA TO PREDICT THE FUNDAMENTAL FREQUENCY OF INITIALLY STRESSED SQUARE PLATES

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(*Received* 23 *October* 2000, *and in final form* 2 *January* 2001)

1. INTRODUCTION

Plates are the widely used structural elements in many areas of engineering disciplines. It is necessary to know their vibration and buckling behaviour to arrive at an efficient design. In actual practice, these plates are initially stressed and it is often necessary to find the vibration characteristics, especially the fundamental frequency of the initially stressed plates.

A good amount of work on the vibration and buckling of plates is compiled in references [1, 2]. However, not much attention is given in reference [1] on the initially stressed vibrations of plates. This is probably due to lack of sufficient work in the literature on this topic at the time of compilation.

The evaluation of fundamental frequency in the presence of initial stresses is rather complex. One has to do repeated analyses, varying the initial stress parameter, which is time consuming, even though the methods are well established at present.

On the other hand, if a simple formula is available to predict the initially stressed frequency once the stress-free frequency and buckling load are known (the stress-free frequency and buckling parameters are routinely evaluated by the designers/analysts), it can be very effectively used by the design engineers in the initial design phase, and this formula avoids repeated complex analysis involved to obtain the stressed frequency.

In this paper, an attempt is made to arrive at a simple formula, which will accurately predict the fundamental frequency of plates. The efficacy of the present formula is demonstrated by applying it to the square plate problem under uniaxial compression solved in reference [3] for three types of boundary conditions and to the square plate problem under biaxial compression for which exact solution is available in reference [1] for simply supported boundary conditions.

The formulation to obtain the formula is briefly presented in the next section.

2. DESIGN FORMULA

For predicting the initially stressed free vibration behaviour of structural elements like bars, beams, plates, etc., following the analyses methods like Rayleigh–Ritz, weighted residual, finite element and so on, the final matrix equation can be obtained in the form

$$[K]\{\delta\} - \lambda[G]\{\delta\} - \lambda_f[M]\{\delta\} = 0, \tag{1}$$

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where in the structural mechanics technology, $[K]$, $[G]$ and $[M]$ may be called as the system stiffness, geometric stiffness and mass matrices, respectively, λ is the compressive load parameter (defined as $\lambda = N_x a^2/D$) and λ_f is the initially stressed frequency parameter (defined as $\lambda_f = \rho\omega^2 a^4/D$) and $\{\delta\}$ is the eigenvector.

From equation (1) the degenerate case of the stability equation is given by

$$[K]\{\delta\} = \lambda_b[G]\{\delta\}, \tag{2}$$

where λ_b is the stability parameter (defined as $\lambda = N_{xcr}a^2/D$). Equation (2) can be written as

$$[G]\{\delta\} = (1/\lambda_b)[K]\{\delta\}. \tag{3}$$

Similarly, a degenerate case for vibration problem from equation (1) can be written as

$$[M]\{\delta\} = (1/\lambda_{f0})[K]\{\delta\}, \tag{4}$$

where λ_{f0} is the stress-free frequency parameter (defined as $\lambda_{f0} = \rho\omega_0^2 a^4/D$).

Assuming that the eigenvectors for the buckling and free vibration are the same as that of the initially stressed vibration of the structural elements under consideration and substituting equations (3) and (4) into equation (1), we get

$$[K]\{\delta\} - \frac{\lambda}{\lambda_b}[K] - \{\delta\}\frac{\lambda_f}{\lambda_{f0}}[K]\{\delta\} = 0. \tag{5}$$

Equation (5) implies that

$$\frac{\lambda}{\lambda_b} + \frac{\lambda_f}{\lambda_{f0}} = 1. \tag{6}$$

From equation (6), knowing λ (compressive load parameter for a particular value of N_x), λ_b and λ_{f0} one can compute the frequency parameter λ_f of the initially stressed structural elements.

3. EXAMPLE PROBLEMS AND DISCUSSION

To examine the effectiveness of the formula, equation (6), in the previous section, two example problems are taken from references [1, 3]. The first problem considered is the prediction of the fundamental frequency parameter of a square plate under uniaxial compression with three types of boundary conditions. The boundary conditions are defined at the edges 1, 2, 3 and 4 as shown in Figure 1, in that order. For example, the boundary conditions if specified as SF CF it means that the edge 1 is simply supported, edges 2 and 4 are free and edge 3 is clamped. In reference [3], a uniaxial compressive load which is 80% of the critical load of SF SF plate is applied for the plates with other boundary conditions of SF CF and CF CF.

For the purpose of demonstrating the formula the fundamental stress-free frequency parameters are taken from reference [1] and the stability parameters are taken from reference [3]. These are required for applying the formula and are given in Table 1 for three types of boundary conditions, namely, SF, SF, SF CF and CF CF.

Table 2 gives the uniaxial load parameter λ applied on the square plate as a percentage of its actual stability parameter (computed by the author) and the initially stressed fundamental frequency parameter obtained from the formula is given along with the

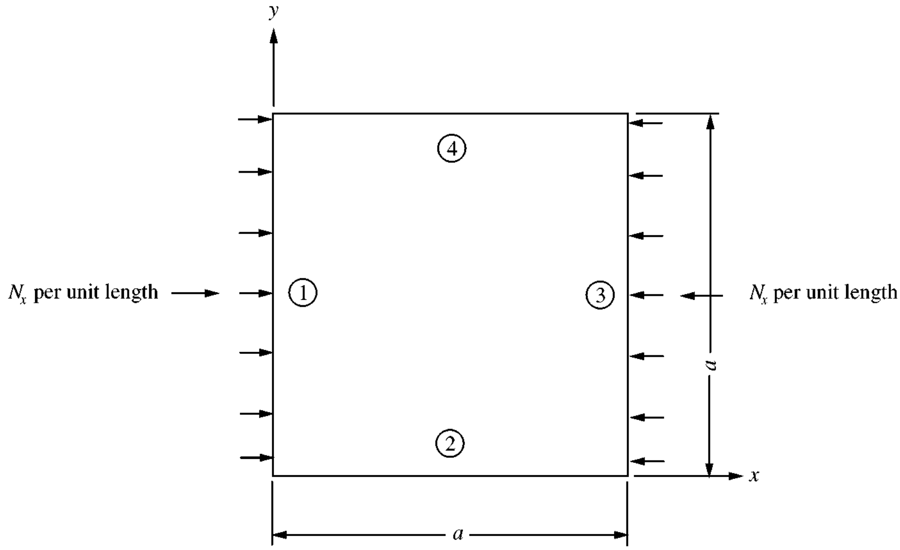

Figure 1. A square plate under uniaxial compression.

Table 1

Stress-free fundamental frequency parameter $(\lambda_{f0})^{1/2}$ and stability parameter λ_b of a square plate under uniaxial compression

Boundary conditions	$(\lambda_{f0})^{1/2}$, reference [1]	λ_b, reference [3]
SF SF	9·636	9·276
SF CF	15·16	19·40
CF CF	22·17	38·45

Table 2

Fundamental frequency parameter $(\lambda_f)^{1/2}$ of a square plate under uniaxial compression

		$(\lambda_f)^{1/2}$		
Boundary conditions	λ/λ_b%	From equation (6)	Reference [3]	% difference
SF SF	80·0	4·307	4·279	0·6544
SF CF	38·25	11·92	11·94	−0·1675
CF CF	19·30	19·92	19·92	0·0

corresponding frequency parameter given in reference [3]. It can be seen that the agreement of the present results is excellent. Also, it is to be noted here that the assumption that the mode shapes for the stress-free vibration, stability and stressed vibration problems is correct. However, when the applied load parameter is 80% of the stability parameter of SF SF plate, the percentage difference of the present result with respect to that given in reference [3] is in error of about 0·65%. This shows that the stress-free and stressed vibration mode shapes and the stability mode shape are very slightly different for this problem. For the other two boundary conditions as the percentage load parameter applied

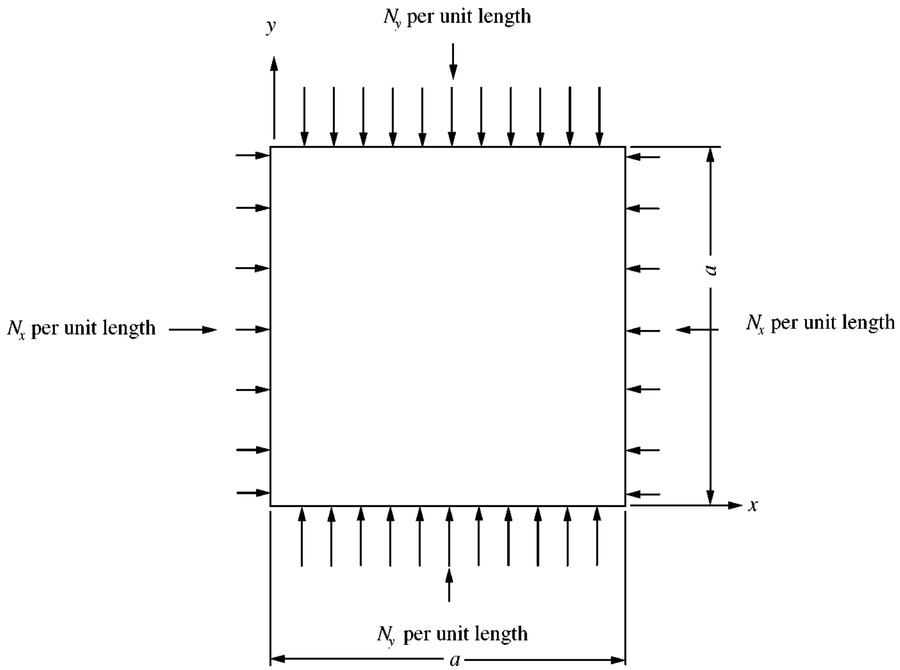

Figure 2. A simply supported square plate under biaxial compression.

is much smaller than the previous case and in view of the excellent agreement of the results it can be said that the assumption made in the previous section is satisfied.

The second problem considered is the stressed vibration of a simply supported square plate under biaxial compression (Figure 2). From reference [1], the frequency parameter λ_f of such a plate is

$$\lambda_f = [(m\pi^2) + (n\pi^2)] - \frac{N_x a^2}{D}(m\pi)^2 + \frac{N_y a^2}{D}(n\pi)^2. \tag{7}$$

For the fundamental mode of vibration ($m = n = 1$) and for uniform biaxial compression ($N_x = N_y$) equation (7) can be written as

$$\lambda_f = 4\pi^4 - \frac{2N_x a^2}{D}\pi^2. \tag{8}$$

Equation (8) can be rewritten in terms of the stability load parameter λ_b, given by

$$\lambda_b = \frac{N_{xcr} a^2}{D} = 2\pi^2 \tag{9}$$

as

$$\lambda_f = \lambda_{f0}\left(1 - \frac{\lambda}{\lambda_b}\right). \tag{10}$$

Equation (9) is exactly same as equation (6) as the three mode shapes for this problem are exactly the same, and hence shows that the present formula derived gives an exact solution for the problem considered and demonstrates its usefulness.

4. CONCLUDING REMARKS

A simple design formula is derived in this paper to obtain the initially stressed frequency parameter of structural elements once the stress frequency parameter, stability parameter and the applied load parameter are available. The usefulness of the formula is demonstrated through the example of a square plate, with different boundary conditions, under uniaxial compression and a simply supported square plate under biaxial compression. The present results validate the assumption made in the formulation. Finally, the author believes that quick and accurate estimates of fundamental frequencies of initially stressed structural elements can be obtained by applying the derived formula and used effectively by the structural designers/analysts.

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APPENDIX A: NOMENCLATURE

a	edge length of a square plate
D	plate flexural rigidity ($= Et^3/12(I - v^2)$)
E	Young's modulus
$[G]$	system geometric stiffness matrix
$[K]$	system elastic stiffness matrix
$[M]$	system mass matrix
N_x	uniaxial compressive load per unit length
N_{xcr}	critical N_x
t	plate thickness
$\{\delta\}$	eigenvector
λ	compressive load parameter ($= N_x a^2/D$)
λ_b	stability load parameter ($= N_{xcr} a^2/D$)
λ_f	stressed frequency parameter ($= \rho\omega^2 a^4/D$)
λ_{f0}	stress free frequency parameter ($= \rho\omega_0^2 a^4/D$)
v	the Poisson ratio
ω	circular frequency of stressed plate
ω_0	circular frequency of stress-free plate
ρ	mass per unit area